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MASS TRANSFER IN ALIGNED-FIELD MAGNETOHYDRODYNAMIC FLOW PAST A FLAT PLATE

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Abstract—Presented herein are boundary-layer studies of the steady mass-transfer phenomena in an aligned-field magnetohydrodynamic flow past a semi-infinite flat plate. The fluid is assumed to be viscous, electrically conducting and incompressible. The plate either is porous such that blowing or suction of a foreign diffusing fluid through the wall surface can be effected, or is made of a material which sublimes into the hot boundary-layer fluid, thus inducing a mass-transfer process. Other parameters being held constant, the skin friction, heat-transfer (except for negative Eckert numbers) and mass-transfer coefficients are decreased by a convective flow away from the wall surface (e.g. blowing or sublimation) and are increased by a convective flow toward the surface (e.g. suction or icing). The effects of the applied magnetic field are such that, for blowing and small suction rates, the magnetic field decreases the skin friction, the induced magnetic field at the wall, the local heat-transfer coefficient and the local mass-transfer coefficient; but at higher suction rates, it increases all the above parameters, except for negative Eckert numbers.

NOMENCLATURE

$A(x, y)$, magnetic stream function such that $\mathbf{H} = \nabla x(A\mathbf{i}_3)$;
C, specific heat;
D, coefficient of diffusion;
Ek, Eckert number $\left[\equiv \dfrac{V_\infty^2}{2C(T_w - T_\infty)}\right]$;
$f(\eta)$, defined as $\Psi(x, y)/[\sqrt{(\nu V_\infty x)}]$;
$g(\eta)$, defined as $A(x, y)/H_0[\sqrt{(\nu x/V_\infty)}]$;
$\mathbf{H}$, magnetic field vector;
H_0, applied magnetic field;
h, local heat-transfer coefficient;
h_D, local mass-transfer coefficient;
$\mathbf{i}_3$, unit vector in z-direction;
l, length of flat plate;
$\dot{m}_1$, mass-flow rate of diffusing fluid;
Nu_x, local Nusselt number ($\equiv hx/\kappa$);
Pm, magnetic Prandtl number ($\equiv \bar{\mu}\sigma\nu$);
Pr, Prandtl number ($\equiv \mu C/\kappa$);
q_w, heat flux density at wall surface;
Re, Reynolds number ($\equiv V_\infty l/\nu$);
Re_x, local Reynolds number ($\equiv V_\infty x/\nu$);
Rm, magnetic Reynolds number ($\equiv \bar{\mu}\sigma V_\infty l$);
S, magnetic force number ($\equiv \bar{\mu}H_0^2/\rho V_\infty^2$);
Sc, Schmidt number ($\equiv \nu/D$);
Sh_x, local Sherwood number ($\equiv h_D x/D$);
T, temperature;
v, V, fluid velocity;
w, mass fraction; ratio of partial density of diffusing component to total density;
x, y, coordinate axes along and normal to flat plate.

Greek symbols

α, thermal diffusivity ($\equiv \kappa/\rho C$);
η, independent variable $[\equiv y/2\sqrt{(V_\infty/\nu x)}]$;
κ, thermal conductivity;
μ, dynamic viscosity;
$\bar{\mu}$, magnetic permeability;
ν, kinematic viscosity ($\equiv \mu/\rho$);
ρ, fluid density;

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σ, electrical conductivity;

$\theta(\eta)$, dimensionless temperature $\left(\equiv \frac{T - T_w}{T_\infty - T_w}\right)$;

$\Phi(\eta)$, dimensionless mass fraction $\left(\equiv \frac{w_1 - w_{1w}}{w_{1\infty} - w_{1w}}\right)$;

$\Psi(x, y)$, fluid stream function such that $\mathbf{v} = \nabla x(\Psi \mathbf{i}_3)$.

Subscripts

∞, free stream;

$0, w$, wall;

1, diffusing fluid;

2, main stream or boundary-layer fluid;

x, y, components in x- and y-coordinate axes.

Superscripts

$', '', '''$, first, second and third derivatives with respect to η except η', η'', and η''' which are dummy variables of η.

INTRODUCTION

THE POSSIBILITY of utilizing a magnetic field by carrying a superconducting magnet on-board a space vehicle to change the reentry trajectories and to combat the reentry heating problem has already been cited [1]. A preliminary heat-transfer analysis shows that the magnetic field effectively increases the thermal boundary-layer thickness [2], thus reinforcing further the concept of using magnetic fields in combating the reentry problem. Another possible solution to the same problem, in the presence of the magnetic field, may be realized with mass-transfer processes. The use of mass-transfer processes or transpiration cooling in maintaining low surface temperatures is quite well-known in classical hydrodynamics [3]. It is thus anticipated that a combination of magneto-hydrodynamic effects and mass-transfer processes would together provide an effective means of shielding and protecting the reentry vehicle from aerodynamic heating.

The problem investigated herein is the aligned flow past a flat plate in which the flow velocity and the magnetic field vectors far from the plate are parallel. Mass transfer is accomplished by injecting or blowing across the fluid–solid boundary into the fluid stream a foreign fluid, whose properties are not too different from those of the free-stream. Such a process can be realized, for example, by the use of porous surfaces or by fabricating the surface of a material which sublimes into the hot boundary-layer fluid. Also included in this analysis is boundary-layer "suction" over a range of suction values. The present study is thus concerned with the prediction of skin friction, heat-transfer and mass-transfer coefficients and temperature and mass-fraction distributions for such mass-transfer (injection or suction) processes.

ANALYSIS

Due to the very complicated nature of this problem, the following statements and assumptions are made prior to the establishment of the governing equations.

(a) A viscous, electrically conducting, incompressible fluid is the working medium for the steady laminar flow. For simplicity, constant physical properties (thermal conductivity κ, kinematic viscosity ν, mass diffusivity D, electrical conductivity σ, magnetic permeability $\bar{\mu}$ and specific heat C) are assumed. In this situation, this means not only that the properties of each component are independent of temperature and pressure, but also the properties of the two components in the fluid differ very little from each other.

(b) The same conservation equations of momentum and magnetic field for zero mass transfer are assumed to be valid in the presence of mass transfer. This implies that any additional momentum and magnetic fluxes associated with mass transfer are negligible.

(c) Effects of thermal diffusion (the Soret effect) and of the reverse process known as the

Dufour effect are neglected, since their contributions are significant only when the temperature and mass concentration gradients are extremely large as in high-speed flows. The fluxes due to these effects are of second order and, in general, are coupled.

(d) The influence of the magnetic forces on the mass-transfer process is negligible to the extent that the nonmagnetic mass-transfer equation can still be used. The mass fraction, however, is affected by the applied magnetic field through the velocity field.

(e) The electric field is taken to be zero since this choice is consistent with the governing Maxwell equations and geometry. This corresponds physically to a short circuited system.

(f) The boundary condition f_0 = constant, which is a necessary condition for the transformation, implies that $V_0\ \alpha/\sqrt{(x)}$. This restriction, however, is justifiable since the "similarity" injection profile is found to agree in most cases with the condition of constant temperature and mass fraction at the fluid–solid boundary [3].

(g) The ratio of the normal length scale of the "inner flow" to that of the "outer flow" is much smaller than unity. As a result of inner–outer flow expansion, the two-dimensional MHD boundary-layer equations are obtained [2, 4].

(h) Other assumptions made are consistent with those of the Greenspan–Carrier flow problem [2, 4].

Invoking the above assumptions, one obtains the following as the leading order boundary-layer flow equations [2, 3, 4]:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0 \tag{2}$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2} + \frac{\bar{\mu}}{\rho} \times \left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_x}{\partial y}\right) \tag{3}$$

$$-\frac{\partial H_x}{\partial y} = \sigma\bar{\mu}(v_x H_y - v_y H_x) \tag{4}$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho\sigma C}\left(\frac{\partial H_x}{\partial y}\right)^2 + \frac{\nu}{C}\left(\frac{\partial v_x}{\partial y}\right)^2 \tag{5}$$

$$v_x \frac{\partial w_1}{\partial x} + v_y \frac{\partial w_1}{\partial y} = D \frac{\partial^2 w_1}{\partial y^2} \tag{6}$$

with the following boundary conditions:

$$\left.\begin{aligned} y = 0:\quad & v_x = 0,\quad v_y = V_0,\quad H_y = 0,\\ & T = T_w,\quad w_1 = w_{1w}\\ y \to \infty:\quad & v_x = V_\infty,\quad H_x = H_0,\\ & T = T_\infty,\quad w_1 = w_{1\infty}. \end{aligned}\right\} \tag{7}$$

If there is no net flow of boundary-layer fluid (or main flow fluid) into the porous wall surface, then $v_2 = 0$ at $y = 0$. This extra boundary condition* enables one to determine the mass fraction of the coolant fluid at the wall w_{1w}, which, in general, is an unknown quantity. By balancing the diffusive flow of boundary-layer fluid into the wall by the convective flow of coolant fluid toward the wall surface such that the net flow of boundary-layer fluid in the wall and through the surface is zero, we get [3]

$$w_{1w} = 1 + \frac{D}{V_0}\left(\frac{\partial w_1}{\partial y}\right)_w. \tag{8}$$

It is thus noted that a definite relationship exists between the mass fraction of the coolant at the wall, w_{1w}, and the rate of "injection" or "suction", V_0.

The nonlinearity of the momentum equation makes difficult a closed mathematical solution to the problem. If, however, one introduces the stream functions $\Psi(x, y)$ and $A(x, y)$, such that

* The nine boundary conditions of equation (7) are sufficient to fully define the problem depicted by equations (1–6).

$\mathbf{v} = \nabla x(\Psi \mathbf{i}_3)$ and $\mathbf{H} = \nabla x(A\mathbf{i}_3)$, as well as the following transformations:

$$\left.\begin{aligned} f(\eta) &= \frac{\psi(x, y)}{\sqrt{(\nu V_\infty x)}} \\ g(\eta) &= \frac{A(x, y)}{H_0\sqrt{(\nu x/V_\infty)}} \\ \theta(\eta) &= \frac{T(x, y) - T_w}{T_\infty - T_w} \\ \phi(\eta) &= \frac{w_1(x, y) - w_{1w}}{w_{1\infty} - w_{1w}} \\ \eta &= \frac{y}{2}\sqrt{(V_\infty/\nu x)} \end{aligned}\right\} \quad (9)$$

the governing equations and their boundary conditions become:

$$f''' + ff'' - Sgg'' = 0 \quad (10)$$

$$g'' + Pm(fg' - gf') = 0 \quad (11)$$

$$\theta'' + (Pr)f\theta' + \frac{(Ek)(Pr)}{2} \times \left[\left(\frac{S}{Pm}\right)(g'')^2 + (f'')^2\right] = 0 \quad (12)$$

$$\phi'' + (Sc)f\phi' = 0 \quad (13)$$

and

$$\left.\begin{aligned} &\eta = 0: \; f' = 0, \quad g = 0, \quad \theta = 0, \quad \phi = 0, \quad f = -2\left(\frac{V_0}{V_\infty}\right)(\sqrt{Re_x}) = f_0 \\ &\eta \to \infty: \; f' = 2, \quad g' = 2, \quad \theta = 1, \quad \phi = 1. \end{aligned}\right\} \quad (14)$$

Here $S \equiv (\bar{\mu}H_0^2)/(\rho V_\infty^2)$, the ratio of the electromagnetic to the inertia forces, is referred to as the magnetic force number; $Pm \equiv \bar{\mu}\sigma\nu$, the ratio of the kinematic viscosity ν to the magnetic viscosity $(1/\bar{\mu}\sigma)$, is the magnetic Prandtl number; $Pr \equiv \mu C/\kappa$, the ratio of the kinematic viscosity ν to the thermal viscosity or diffusivity α, is the Prandtl number; $Ek \equiv V_\infty^2/[2C(T_w - T_\infty)]$ is the Eckert number; and $Sc \equiv \nu/D$ is the Schmidt number. It may be noted tnat $f_0 < 0$ indicates "injection" or "blowing" of a foreign or coolant fluid across the porous wall surface into the fluid main stream, whereas $f_0 > 0$ depicts "suction" from the boundary-layer fluid or mass transfer toward the wall which could occur, for example, in the case of icing. It is further noted that the applied magnetic field destroys the analogous profiles of the velocity, temperature, and mass fraction. Only in the nonmagnetic case ($S = 0$), where, in addition, negligible frictional heating and $Pr = Sc = 1$ are assumed, are the three profiles identical. Analogous profiles exist between the temperature and the mass fraction as long as $Pr = Sc$ in a nonmagnetic, negligible viscous heating flow.

The second order inhomogeneous linear differential equation for the temperature parameter $\theta(\eta)$, equation (12), which describes heat transfer in a laminar steady boundary-layer flow with ohmic and viscous heating, can be integrated, for instance, by the method of variation of coefficients to yield

$$\theta(\eta) = \left[1 + \frac{Ek\,Pr}{2}\int_0^\infty \exp\left(-\int_0^{\eta'} Pr\,f\,\mathrm{d}\eta''\right)\mathrm{d}\eta' \int_0^{\eta'} \left[(f'')^2 + \frac{S}{Pm}(g'')^2\right]\exp\left(\int_0^{\eta''} Pr\,f\,\mathrm{d}\eta'''\right)\mathrm{d}\eta''\right]$$
$$\times \frac{\int_0^\eta \exp(-\int_0^{\eta'} Pr\,f\,\mathrm{d}\eta'')\,\mathrm{d}\eta'}{\int_0^\infty \exp(-\int_0^{\eta'} Pr\,f\,\mathrm{d}\eta'')\,\mathrm{d}\eta'} - \frac{(Ek)(Pr)}{2}$$

$$\times \int_0^\eta \exp\left(-\int_0^{\eta'} Prf\,d\eta''\right) d\eta' \int_0^{\eta'} \left[(f'')^2 + \frac{S}{Pm}(g'')^2\right] \exp\left(\int_0^{\eta''} Prf\,d\eta'''\right) d\eta''. \qquad (15)$$

Its slope at the wall, $\theta'(0)$, is

$$\theta'(0) = \frac{1 + \frac{(Ek)(Pr)}{2}\int_0^\infty \exp\left(-\int_0^{\eta'} Prf\,d\eta''\right) d\eta' \int_0^{\eta'} \left[(f'')^2 + \frac{S}{Pm}(g'')^2\right] \exp\left(\int_0^{\eta''} Prf\,d\eta'''\right) d\eta''}{\int_0^\infty \exp\left(-\int_0^{\eta'} Prf\,d\eta''\right) d\eta'}. \qquad (16)$$

The rate of heat transfer per unit area at the wall is then given by:

$$q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_w = \frac{\kappa}{2}\sqrt{(V_\infty/\nu x)}\,(T_w - T_\infty)\,\theta'(0). \qquad (17)$$

Introducing Newton's law of cooling as $q_w = h(T_w - T_\infty)$, where h is the heat-transfer coefficient, one can readily equate this to equation (17) and form a dimensionless local Nusselt number as

$$Nu_x \equiv \frac{hx}{\kappa} = \frac{\theta'(0)}{2}\sqrt{(Re_x)}. \qquad (18)$$

Of particular interest is the ratio of the heat-transfer coefficient with mass transfer to that of zero blowing or suction value. This ratio is related to the corresponding temperature parameter gradients in the following manner:

$$\frac{h}{h_{f_0=0}} = \frac{\theta'(0)}{[\theta'(0)]_{f_0=0}}. \qquad (19)$$

Direct integration of the mass-transfer equation (13) results in

$$\phi(\eta) = \frac{\int_0^\eta \exp(-\int_0^{\eta'} Scf\,d\eta'')\,d\eta'}{\int_0^\infty \exp(-\int_0^{\eta'} Scf\,d\eta'')\,d\eta'} \qquad (20)$$

and

$$\phi'(0) = \frac{1}{\int_0^\infty \exp(-\int_0^{\eta'} Scf\,d\eta'')\,d\eta'}. \qquad (21)$$

The mass flow of the diffusing fluid at the wall surface is given by

$$\dot{m}_{1w} = -\frac{\rho D}{(1 - w_{1w})}\left(\frac{\partial w_1}{\partial y}\right)_w = \rho\left(\frac{D}{2}\right) \times \sqrt{(V_\infty/\nu x)}\,\frac{(w_{1w} - w_{1\infty})}{(1 - w_{1w})}\,\phi'(0) \qquad (22)$$

which, with the introduction of a mass-transfer coefficient h_D such that

$$\dot{m}_{1w} = \rho h_D \frac{(w_{1w} - w_{1\infty})}{(1 - w_{1w})},$$

yields a dimensionless local Sherwood number as

$$Sh_x \equiv \frac{h_D x}{D} = \frac{\phi'(0)}{2}(\sqrt{Re_x}). \qquad (23)$$

The ratio of the mass-transfer coefficient in the presence of mass transfer at the wall to that of zero blowing or suction value is then given by

$$\frac{h_D}{(h_D)_{f_0=0}} = \frac{\phi'(0)}{[\phi'(0)]_{f_0=0}}. \qquad (24)$$

In terms of the new parameters, equation (8) can be rewritten as

$$\frac{1 - w_{1w}}{w_{1\infty} - w_{1w}} = \frac{\phi'(0)}{Scf_0} \qquad (25)$$

thus relating the blowing or suction value f_0 to the values of w_{1w} and $w_{1\infty}$ of the mass fraction of the diffusing fluid at the wall surface and in the free stream, i.e. only two of the three values can be prescribed arbitrarily. If $w_{1\infty} = 0$, as is generally the case in transpiration cooling, the mass fraction of the diffusing fluid at the wall surface is then related to the blowing value according to

$$w_{1w} = \frac{1}{1 - \frac{\phi'(0)}{Scf_0}}. \tag{25a}$$

In this analysis, equations (10) and (11) are solved simultaneously, subject to the boundary conditions on f and g depicted by equation (14). The IBM 7094 computer is used for the numerical computation. Values of f and g and their derivatives for each value of η are then used in numerically integrating equations (15, 16, 20, 21). The ranges of S and Pm have been assigned such that they are consistent with the existence of a steady-state solution, i.e. $0 \leqslant S \leqslant 1$, and with the validity of the boundary-layer equations. For brevity, only a portion of the representative results are given.

RESULTS

The dependence of the skin friction, $f''(0)$, on the blowing and suction values f_0 is shown in Figs. 1(a, b) for $Pm = 0{\cdot}1$ and $0{\cdot}6$. In general, the skin friction increases with increasing suction values but decreases with increasing blowing rates to a zero value. For all blowing values and small suction values, the skin friction is decreased with increasing magnetic field; whereas, for large suction values, larger values of skin friction can be effected by increasing the applied field. An argument which rationalizes the above conclusion requires a discussion of the balance among the various forces present. There are effectively three forces—the Lorentz force, the induced pressure force and the mechanical force due to either blowing or suction process—which are competing with one another at the fluid–solid boundaries. In the absence of mass transfer across the plate, i.e. $f_0 = 0$, it has been established that the induced pressure force opposes and dominates the Lorentz force, yielding a resulting force directed away from the plate and tending to thicken the boundary-layer thickness and decrease the skin friction [4]. In blowing, the additional mechanical force due to the blowing process aids and strengthens the induced pressure force against the Lorentz force, yielding, therefore, smaller values of skin friction as the blowing rate is increased. At large suction rates, e.g. $f_0 \gg 1$, the induced pressure force and the mechanical force of suction are directed toward the plate, while the Lorentz force is in the opposite direction. Therefore, the sum of the mechanical suction force and the induced pressure force is more than the Lorentz force, giving a resultant force directed toward the plate, and in turn, yielding a thinner boundary-layer thickness and a larger value of skin friction. As one crosses from the blowing to the suction process, however, one notes that the "pull" of the magnetic lines toward the plate by the suction process is beginning to offset the "lift" of these lines due to the upward tilting of the stream lines in the presence of the plate, and the induced pressure force is beginning to reverse its direction. The Lorentz force is now directed away from the plate. For small suction values, therefore, the Lorentz force, aided vectorially by the induced pressure force, still dominates over the mechanical suction force, yielding results depicted in Fig. 1 for $0 < f_0 < 1$. The transition value or range of values within which $f''(0)$ is unaltered by the magnetic field is a function of the magnetic Prandtl number, being at smaller values of f_0 for larger Pm values, e.g. $f_0 \simeq 0{\cdot}95$ at $Pm = 0{\cdot}1$ and $f_0 \simeq 0{\cdot}86$ at $Pm = 0{\cdot}6$. For the nonmagnetic case, the boundary-layer thickness becomes infinite at and beyond a blowing value of $f_0 = -1{\cdot}238$ [3]. In the presence of the applied field, however, it becomes infinite at smaller blowing values. It should be cautioned that, when this happens or even before it happens, the boundary-layer analysis on which this study is based no

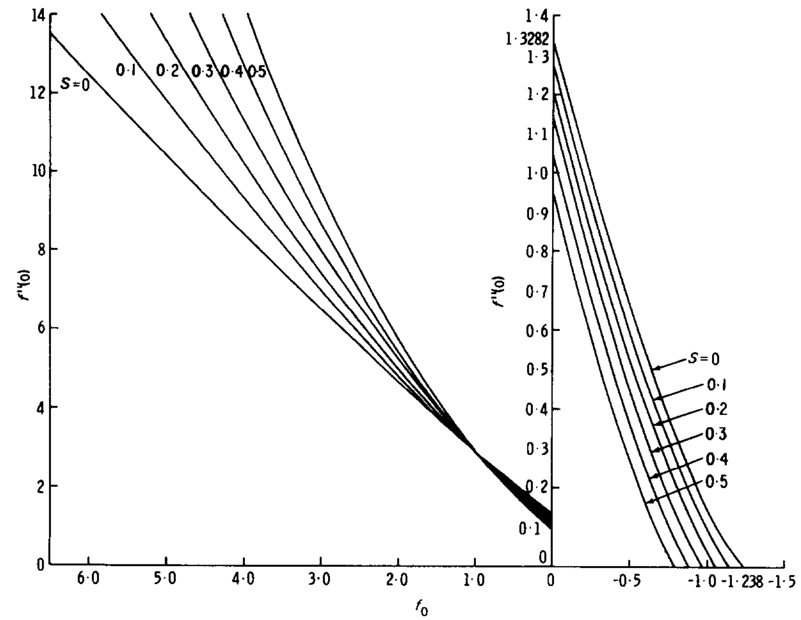

FIG. 1(a). "Skin friction" as a function of blowing and suction values for $Pm = 0{\cdot}1$.

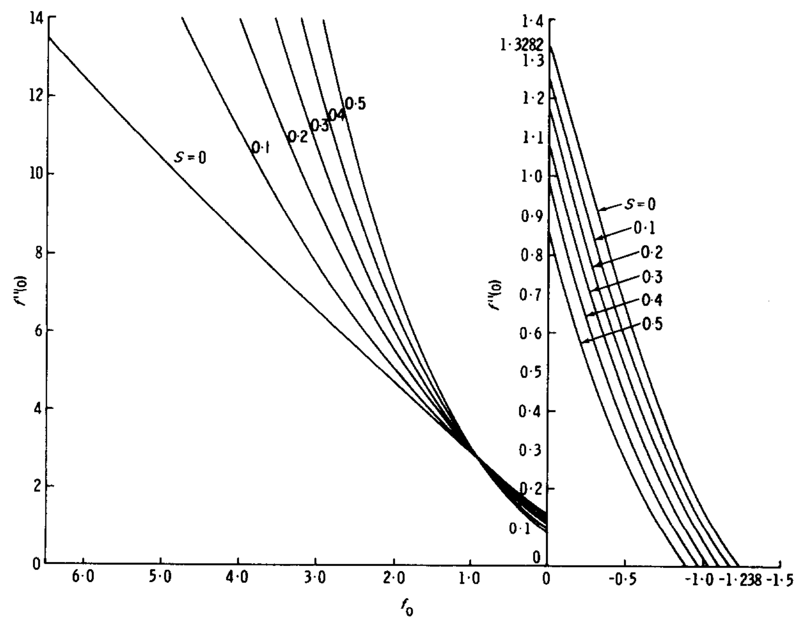

FIG. 1(b). "Skin friction" as a function of blowing and suction values for $Pm = 0{\cdot}6$.

longer describes the actual phenomena taking place.

The induced magnetic field at the wall, $g'(0)$, is shown in Fig. 2. It is observed that the induced field exhibits similar trends as $f''(0)$ in its relation with the magnetic force number and the blowing and suction values. The remarks made on $f''(0)$ can literally be applied here to $g'(0)$ if, instead of the forces, induced currents are discussed.

The dimensionless temperature and mass-fraction profiles for the nonmagnetic boundary-layer flow are shown in Fig. 3 over a range of blowing and suction values and for two $Pr = Sc$ values, 0·7 and 1·0. It is noted that, for the case of negligible viscous heating, these curves are independent of the Eckert number, as can be seen from equation (12) when $S = 0$. The $Pr = Sc = 1{\cdot}0$ curves are then identical to the dimensionless velocity profiles for the non-magnetic case. These results have also been obtained by Hartnett and Eckert [3].

The dependence of the dimensionless temperature on the Eckert number can be seen from Fig. 4. For the range of blowing and suction values considered, the temperature for $Ek = 1{\cdot}0$ is always greater than that for $Ek = -1{\cdot}0$. As the suction rate is increased, the temperature is increased or decreased depending on whether $Ek = 1{\cdot}0$ or $Ek = -1{\cdot}0$, respectively. Comparing the results of Fig. 4 with Fig. 3, it is seen that for blowing and for small values of suction, the applied magnetic field decreases the temperature of the fluid close to wall surface, but increases that near the outer boundary. For large suction values, the temperature of the boundary-layer fluid is greatly increased by the applied field. The dependence of the temperature on the Prandtl number is similar to the trend depicted by Fig. 3 in that, for suction and small blowing values, the temperature profiles at $Pr = 0{\cdot}7$ have a smaller initial slope and are always below those at $Pr = 1{\cdot}0$, therefore yielding thicker thermal-boundary layer for the lower Prandtl number. At higher blowing values, however, the initial slope is smaller at larger

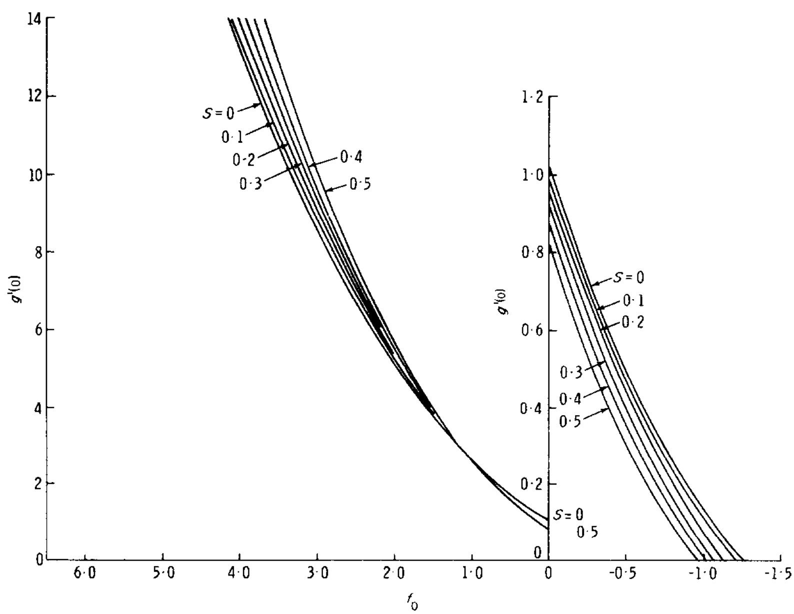

FIG. 2. Induced magnetic field at the wall as a function of blowing and suction values ($Pm = 0{\cdot}6$).

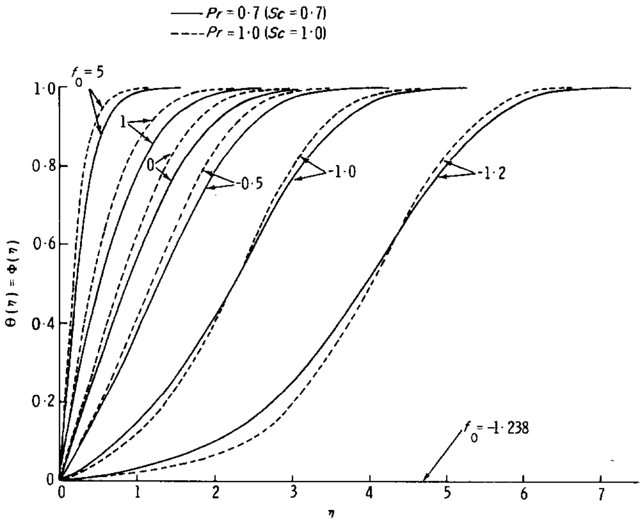

FIG. 3. Dimensionless temperature and mass-fraction profiles for nonmagnetic boundary-layer flow.

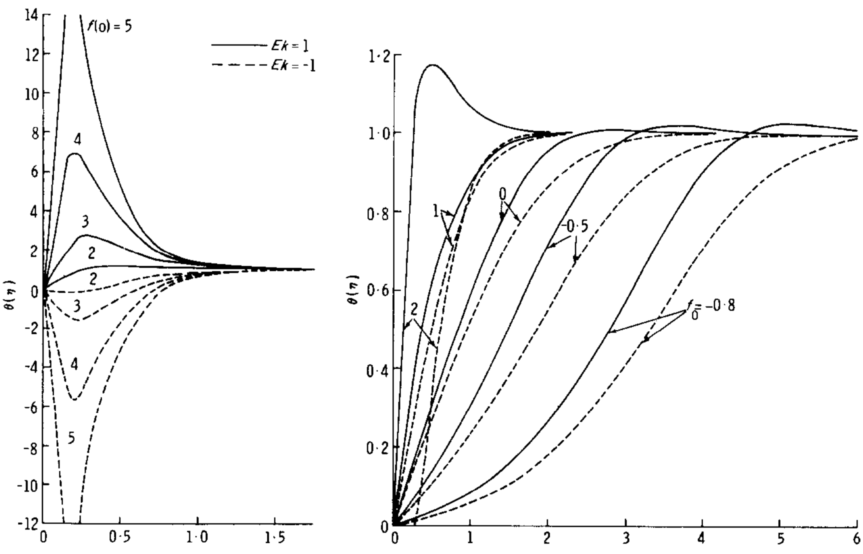

FIG. 4. Dimensionless temperature profiles for laminar flow over flat plate for a range of blowing and suction values (Pm = 0·6, Pr = 1·0, S = 0·5).

Prandtl number, but the temperature profile for $Pr = 0{\cdot}7$ crosses over that at $Pr = 1{\cdot}0$, again yielding thicker thermal-boundary layer for the lower Prandtl numbers.

The ratio of the heat-transfer coefficients depicted by equation (19) is shown in Figs. 5(a, b) for $Pm = 0{\cdot}6$ and $Pr = 0{\cdot}7$ and 1·0. For the entire range of blowing and suction values, $\theta'(0) > 0$ for $Ek = 1{\cdot}0$. In the case of $Ek = -1{\cdot}0$, however, $\theta'(0) > 0$ only for blowing and for small suction values. In general, the applied magnetic field decreases the heat-transfer co-

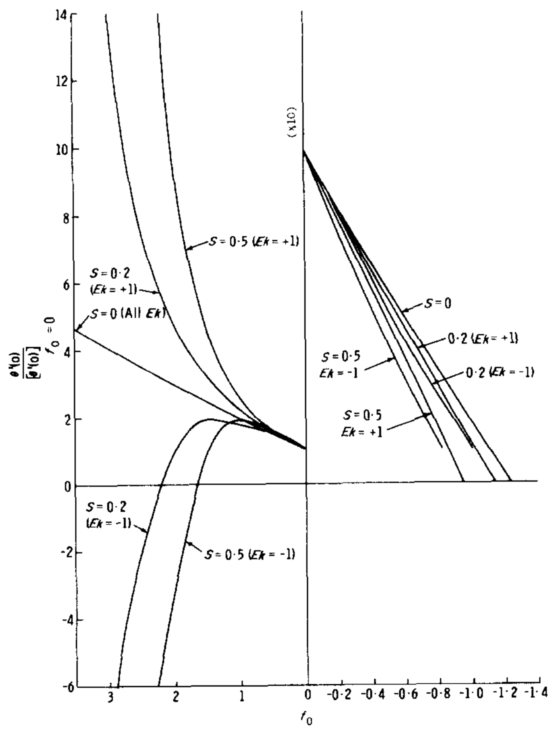

FIG. 5(a). Ratio of heat-transfer coefficients as a function of blowing and suction values ($Pm = 0{\cdot}6$, $Pr = 0{\cdot}7$).

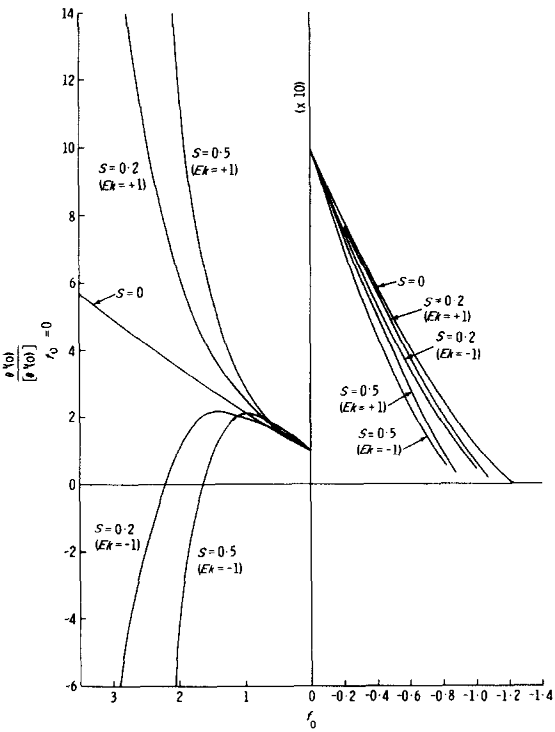

FIG. 5(b). Ratio of heat-transfer coefficients as a function of blowing and suction values ($Pm = 0{\cdot}6$, $Pr = 1{\cdot}0$).

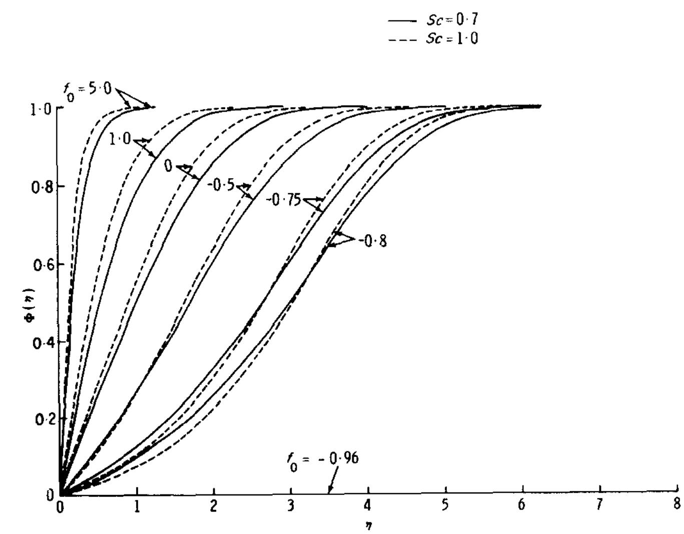

FIG. 6. Dimensionless mass-fraction profiles over a range of blowing and suction values ($Pm = 0{\cdot}6$, $S = 0{\cdot}5$)

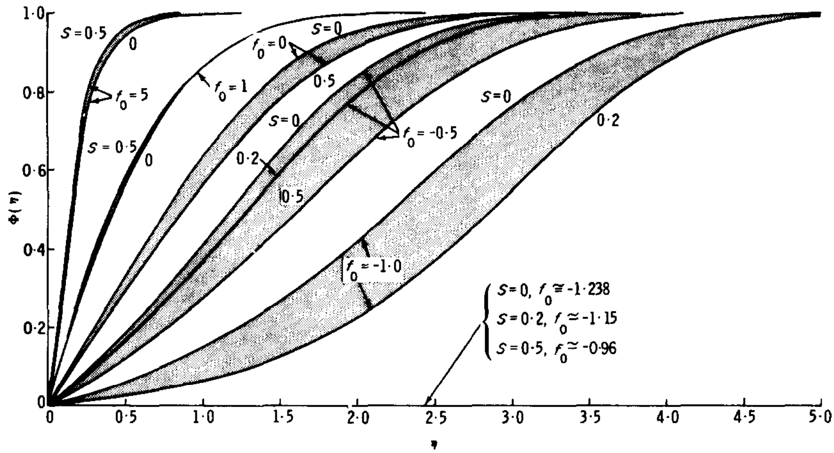

FIG. 7. Dimensionless mass-transfer profiles over a range of blowing and suction values and for a range of magnetic force numbers ($Pm = 0{\cdot}6$, $Sc = 1{\cdot}0$).

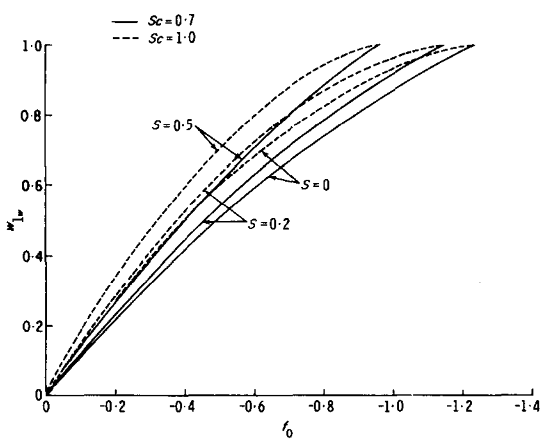

FIG. 8. Mass fraction of diffusing fluid at the wall surface in blowing ($Pm = 0{\cdot}6$, $w_{1\infty} = 0$).

efficients for the same blowing condition, irrespective of whether $Ek = 1{\cdot}0$ or $Ek = -1{\cdot}0$. For the same suction condition, however, the applied field increases the heat-transfer coefficient for $Ek = 1{\cdot}0$, whereas for $Ek = -1{\cdot}0$, it increases the heat-transfer coefficient at small suction values but decreases it at higher suction values.

Figures 6 and 7 depict the dimensionless mass-fraction profiles over a range of blowing and suction values. The remarks made on the Prandtl number to the dimensionless temperature profiles previously also apply here except that now it is the Schmidt number to the dimensionless mass-fraction profiles. Note also the similarity in profiles between Figs. 3 and 6, only that for $S = 0{\cdot}5$ and $Pm = 0{\cdot}6$, the boundary-layer thickness is "blown off"* at $f_0 \simeq -0{\cdot}96$ compared to $f_0 \simeq -1{\cdot}238$ for the non-magnetic case. For blowing and for small suction values, the applied field decreases the mass fraction within the boundary layer, thus increasing the concentration-boundary-layer thickness. At higher suction rates, the applied field increases the dimensionless mass fraction.

The mass fraction of the diffusing fluid at the porous wall surface for the case of $w_{1\infty} = 0$, as is the case in transpiration cooling, is shown in Fig. 8 over a range of blowing values. The wall mass fraction is greater at $Sc = 0{\cdot}7$ than at $Sc = 1{\cdot}0$, and is also greater at higher magnetic force numbers.

The ratio of the mass-transfer coefficients depicted by equation (24) is shown in Figs. 9(a, b) for two values of magnetic Prandtl number, 0·1 and 0·6. The applied magnetic field

* The expression "blown off" depicts a condition in which $\partial v_x/\partial y = 0$ at $y = 0$ and $v_x = 0$ for any finite y.

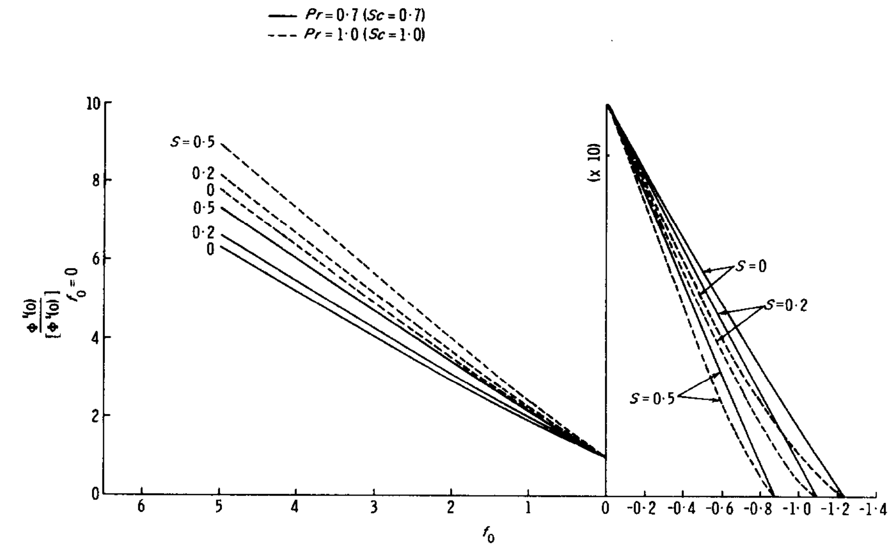

FIG. 9(a). Ratio of mass-transfer coefficients as a function of blowing and suction values ($Pm = 0{\cdot}1$).

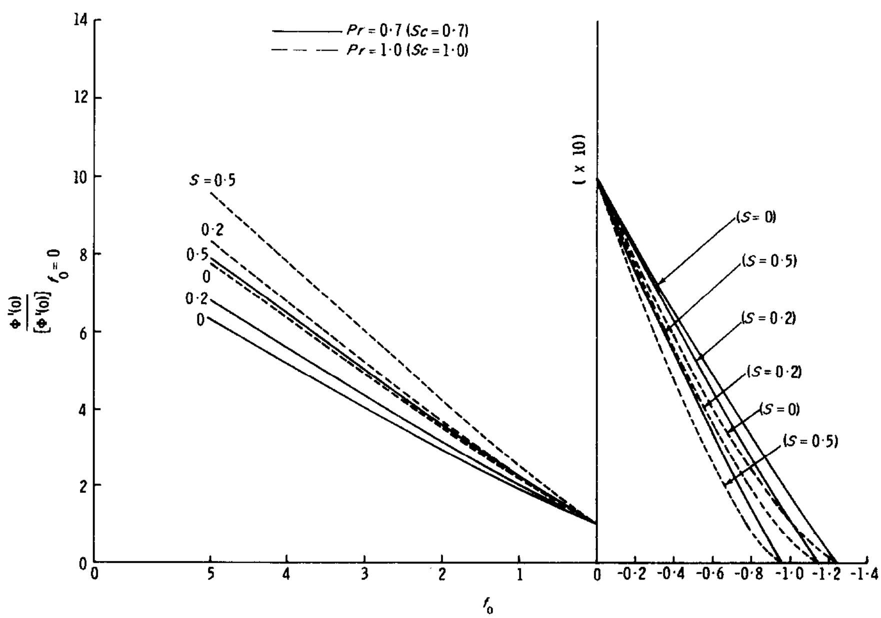

FIG. 9(b). Ratio of mass-transfer coefficients as a function of blowing and suction values ($Pm = 0{\cdot}6$).

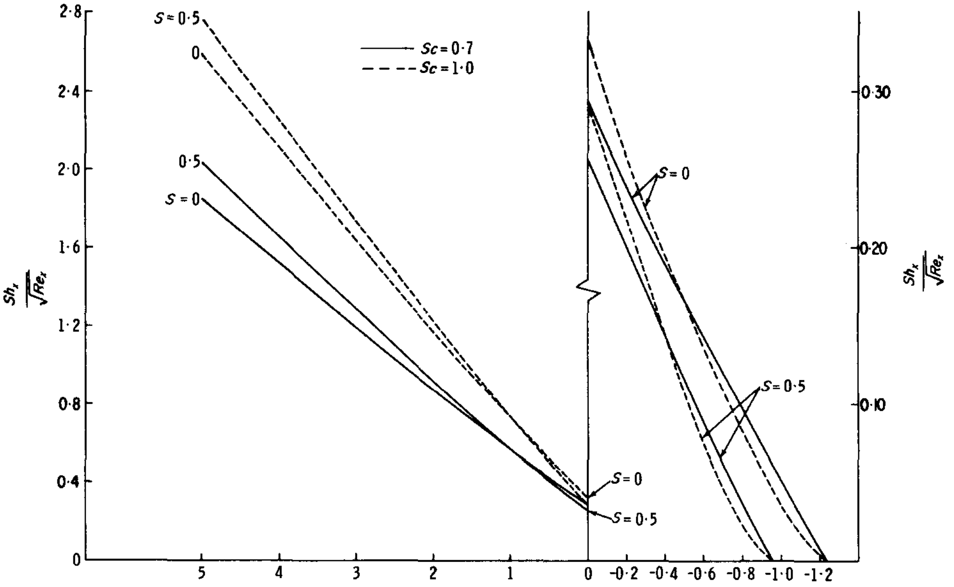

FIG. 10. Dimensionless local mass-transfer coefficients $Sh_x/(\sqrt{Re_x})$ as a function of blowing and suction values ($Pm = 0{\cdot}6$).

increases the mass-transfer coefficient in the case of suction, but decreases it in mass blowing. The blowing rate required to completely "blow off" the boundary layer is smaller at smaller magnetic Prandtl number except in the nonmagnetic case, whereupon the value of Pm does not matter. For example, taking $S = 0{\cdot}2$ and $0{\cdot}5$, the blowing values corresponding to these S values such that the boundary layer becomes infinite are $f_0 \simeq -1{\cdot}09$ and $-0{\cdot}88$, respectively, for $Pm = 0{\cdot}1$, but $f_0 \simeq -1{\cdot}15$ and $-0{\cdot}96$, respectively, for $Pm = 0{\cdot}6$. The $S = 0$ curves are identical to the corresponding curves shown in Figs. 5(a, b).

The dimensionless local mass-transfer coefficient, $Sh_x/(\sqrt{Re_x})$, is shown in Fig. 10 for $S = 0$, $0{\cdot}5$ and $Sc = 0{\cdot}7$, $1{\cdot}0$. For the nonmagnetic case, the $Sh_x/(\sqrt{Re_x})$ curves correspond to those given in [3]. The value of $Sh_x/(\sqrt{Re_x})$ is decreased by blowing and increased by suction.

It is thus concluded that, other parameters being held constant, the skin friction, heat-transfer (except for negative Eckert numbers) and mass-transfer coefficients are decreased by a convective flow away from the wall surface (e.g. injection or blowing) and are increased by a convective flow toward the surface (e.g. suction or icing). The effects of the applied magnetic field are that, for blowing and for small suction values, the magnetic force number S decreases the skin friction $f''(0)$, the induced magnetic field at the wall $g'(0)$, the dimensionless local heat-transfer coefficient $\theta'(0)$ and the dimensionless local mass-transfer coefficient $\phi'(0)$; but at higher suction rates, S increases all the above parameters, except for $Ek < 0$, where $\theta'(0)$ is decreased by increasing S.

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Résumé—On présente ici l'étude de la couche limite dans les phénomènes de transport de masse permanents lorsqu'on a un écoulement magnétohydrodynamique à champ magnétique collinéaire avec la vitesse le long d'une plaque plane semi-infinie. Le fluide est supposé être visqueux, conducteur de l'électricité et incompressible. La plaque est soit poreuse de telle façon qu'on peut souffler ou aspirer un fluide étranger diffusant à travers la surface de la paroi, ou faite d'un matériau qui se sublime dans le fluide chaud de la couche limite, provoquant ainsi un processus de transport de masse. Les autres paramètres étant maintenus constants, le frottement pariétal, les coefficients de transport de chaleur (sauf pour des nombres d'Eckert négatifs) et de transport de masse sont diminués par un écoulement de convection loin de la surface de la paroi (par exemple, soufflage ou sublimation) et sont augmentés par un écoulement de convection vers la surface (par exemple, aspiration ou givrage). Les effets du champ magnétique sont tels que, pour le soufflage et de faibles vitesses d'aspiration, le champ magnétique diminue le frottement pariétal, le champ magnétique induit à la paroi, le coefficient local de transport de chaleur et le coefficient local de transport de masse; mais à des vitesses d'aspiration plus élevées, il augmente tous les paramètres cités ci-dessus, sauf pour des nombres d'Eckert négatifs.

Zusammenfassung—Es werden Grenzschichtuntersuchungen wiedergegeben für den stationären Stoffübergang in einem ausgerichteten magnetohydrodynamischen Strömungsfeld entlang einer halbunendlichen ebenen Platte. Das Medium ist als zäh, elektrisch leitend und inkompressibel angenommen. Die Platte ist entweder porös, so dass Einblasen oder Absaugen eines anderen diffundierenden Stoffes durch die Oberfläche möglich ist, oder aus einem, in das heisse Grenzschichtmedium sublimierenden Stoff hergestellt, wodurch ebenfalls ein Stoffübergang bewirkt wird. Mit sonst konstant gehaltenen Parametern werden die Reibung, die Wärmeübergangskoeffizienten (mit der Ausnahme für negative Eckert-Zahlen) und die Stoffübergangskoeffizienten herabgesetzt durch einen von der Wand weggerichteten Konvektionsstrom (d.h. bei Ausblasung und Sublimation) und sie werden erhöht durch einen zur Wand hingerichteten Konvektionsstrom (d.h. bei Absaugung oder Verfestigung). Das aufgebrachte Magnetfeld bewirkt beim Ausblasen und geringer Absaugung eine Abnahme der Wandreibung des induzierten Magnetfeldes an der Wand, des örtlichen Wärmeübergangskoeffizienten und des örtlichen Stoffübergangskoeffizienten; bei grosser Absaugung jedoch vergrössern sich alle oben erwähnten Parameter, mit Ausnahme jenes für negative Eckert-Zahlen.

Аннотация—Изложены результаты исследования стационарного массообмена при обтекании полубесконечной плоской пластины магнитогидродинамическим потоком в присутствии продольного магнитного поля. Жидкость считается вязкой, электропроводной и несжимаемой. Пластина является либо пористой, так что через поверхность стенки можно осуществить вдув или отсос инородной диффундирующей жидкости, либо изготовлятся из материала, который сублимирует в горячий пограничный слой, что вызывает процесс переноса массы. Все параметры являются постоянными, за исключением коэффициентов поверхностного трения, теплообмена (кроме случая отрицательных чисел Эккерта) и массообмена, которые уменьшаются под действием конвективного потока по направлеиню от стенки (например, вдув или сублимация) и увеличиваются под влиянием конвективного потока к поверхности (например, отсос или обледенение). При вдуве и небольших скоростях отсоса внешнее магнитное поле уменьшает поверхностное трение, индуцированное магнитное поле на стенке, локальный коэффициент теплообмена и локальный коэффициент массообмена. Однако при больших скоростях отсоса, магнитное поле увеличивает все вышеуказанные параметры, кроме случая отрицательных чисел Эккерта.